NAME: _____ **Math2701-001**

Exam 2: Sections $2.4 - 2.6, 4.1 - 4.2$ (5^{th} ed.); $3.4 - 3.7, 5.1 - 5.2$ (6^{th} ed.) Discrete Structures

Please show all work. When some work is required, **NO WORK = NO CREDIT.**

1. (6 points) What sequence of pseudorandom numbers is generated using the linear congruential generator $x_{n+1} = 3x_n + 7 \mod 13$ with seed $x_0 = 2$?

2. (5 points) We can expect how many prime numbers
   (a) in the interval from 1 to 10^6?

   (b) in the interval from 1 to 10^9?

   (c) Based on your previous two answers, we can expect there to be how many prime numbers in the interval from 10^6 to 10^9?

3. (8 points) Books are identified by an International Standard Book Number (ISBN), a 10-digit code $x_1x_2\ldots x_{10}$ assigned by the publisher. These 10 digits consist of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally, a 1-digit check digit that is either a digit or the letter X (used to represent 10). This check digit selected so that $\sum_{i=1}^{10} ix_i \equiv 0 (\text{mod } 11)$ and is used to detect errors in individual digits and transposition of digits.

   The first 9 digits of the ISBN of Melvyn B. Nathanson's *Additive Number Theory* are $0-387-94655$. What is the 10^{th} digit (the check digit) of the ISBN of this book?

4. (4 points) Convert $(DAD)_{16}$ from its hexadecimal expansion to its decimal (base 10) expansion.

5. (4 points) Convert 11 from its decimal (base 10) expansion to its binary (base 2) expansion.

6. (10 points) The **Euler ϕ-function** is defined as

$$\phi(n) := \text{ the number of positive integers } \leq n \text{ that are relatively prime to } n.$$

Find each of the following:

(a) $\phi(3) =$

(b) $\phi(4) =$

(c) $\phi(6) =$

(d) $\phi(8) =$

(e) $\phi(24) =$

Extra Credit(3 points)
Note that $3 \cdot 8 = 24 = 4 \cdot 6$. What are $\phi(3) \cdot \phi(8)$ and $\phi(4) \cdot \phi(6)$? Compare these with $\phi(24)$ and form a conjecture.

7. (14 points)

(a) What is $\gcd(14, 25)$?

(b) Write the g.c.d. of 14 and 25 as a linear combination of 14 and 25.

8. (10 points) Use Algorithm 5 to find $66^{11} (\mathrm{mod} 99)$.

9. (8 points) Solve

$$14x \equiv 9(\text{mod}25).$$

10. (EXTRA CREDIT 2 points) Solve

$$14x + 7 \equiv 16(\text{mod}25).$$

11. (15 points) Find the solutions to the system

$$x \equiv 1(\text{mod}4)$$

$$x \equiv 2(\text{mod}5)$$

$$x \equiv 3(\text{mod}7).$$

12. (4 points) How many license plates can be made using either two letters followed by four digits or two digits followed by four letters?

13. (4 points) How many integers between 1 and 100 are divisible by both 4 and 5?

14. (4 points) How many one-to-one functions are there from a set with 4 elements to
    (a) a set with 3 elements?

    (b) a set with 5 elements?

15. (4 points) Show that if there are 27 students in a class, then at least two have last names that start with the same letter.